Department of Mathematics, Statistics and Computer Science
presents
Properties of the Commuting Graph of the Symmetric Inverse Semigroup
by
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The commuting graph of a finite non-commutative semigroup $S$, denoted $\mathcal{G}(S)$, is a simple graph whose vertices are the non-central elements of $S$ and two distinct vertices $x, y$ are adjacent if $xy = yx$. This definition generalizes the corresponding concept of the commuting graph of a non-Abelian group.

Our work looks at the commuting graph of the symmetric inverse semigroup $\mathcal{I}(X)$. For a finite set $X$, let $\mathcal{I}(X)$ be the semigroup of all partial injective transformations on $X$ under composition. The semigroup is universal for the class of inverse semigroups in the sense that every inverse semigroup can be embedded in $\mathcal{I}(X)$ for some finite set $X$, analog to the situation of the symmetric groups $\text{Sym}(X)$ in group theory.

In 1989, Burns and Goldsmith classified the maximum order abelian subgroups of $\text{Sym}(X)$. We extend this result to the semigroup $\mathcal{I}(X)$. As a consequence, we obtain a formula for the clique number of the commuting graph of $\mathcal{I}(X)$. We also calculate the diameter of $\mathcal{I}(X)$ when $|X|$ is prime or even, and obtain tight bounds on it in the remaining cases.

Refreshments will be served before the talk in Annex 24A